concluded that when the data are expressed as in Equation (5) or (7), the wall effect is a function of the (a/R) ratio only. Up to sphere-to-cylinder-diameter ratios of 0.3125 and eccentricities of 50% of the cylinder radius the effect of the cylindrical boundary is not affected by an increase of the Reynolds number and by various positions of eccentricity. It is the same as derived by the creeping-motion equations.

The present work does not agree with the theory of Faxen, who uses the Oseen approximation (9), where the value of $L|\delta L|$ in Equation (3) decreases with an increase of the Reynolds number.

ACKNOWLEDGMENT

The authors wish to thank the Texas Company for supporting this research through a fellowship.

NOTATION

a = radius of sphere
b = cylinder axis to center of sphere distance
C = actual drag coefficient of a sphere in a bounded medium $= W/[\rho u_o^2/2)\pi a^2]$
C_A = actual drag coefficient of a sphere in an unbounded medium
C_S = drag coefficient according to Stokes's law equal to $24/N_{Re_o}$
$[(C_A/C_S)-1]$ = dimensionless group in Equations (5) and (7), indicating fractional deviation of actual drag from drag calculated by Stokes's law
d = diameter of sphere
D = diameter of cylinder
$F(\beta)$ = function in Equations (4), (4a), and (4b) defined by $(2.105 - 0.6977\ \beta^2)$
K = constant in Equation (5)
N_{Re_o} = Reynolds number based on approach velocity to the sphere $= [(d\,u_o\,\rho/\mu)](1-\beta^2)$
R = radius of cylinder
u_o = fluid velocity at cylinder axis
u = velocity of sphere
W = drag force on the sphere in a bounded medium
W_A = drag force on a sphere in an unbounded medium
β = dimensionless number equal to b/R
μ = viscosity of fluid
ρ = density of fluid

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Centrifugal Filtration Through Beds of Small Spheres

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In 1856 Darcy observed that water flowed through a sand bed at a rate proportional to the area of the bed and to the difference between the upstream and dowstream hydraulic heads and inversely proportional to the bed thickness. Since that time many investigators have accumulated masses of data in support of numerous theoretical and empirical relationships for correlating and predicting pressure drop, flow rate, and fluid and bed properties. Naturally certain areas of research have received less attention than others.

Among the areas receiving least attention are flow through beds composed of particles of identical shape but mixed size, flow through unconsolidated beds composed of small particles, and flow under the influence of a centrifugal driving force. Accordingly, the flow of liquids through unconsolidated beds of small spherical particles of mixed size was investigated in two laboratory centrifugal filters. Experimental conditions were such that the flow was laminar and the cakes were incompressible. While there was some difficulty in obtaining reproducibility of cakes, the data for any particular cake were correlated satisfactorily by the Darcy equation adapted for centrifugal filtration.

THEORY

A mathematical expression for laminar flow of an incompressible fluid in a centrifugal filter may be derived from the classical Darcy equation, which is written

$$\frac{\Delta P}{L} = \frac{u\mu}{\alpha g_c} \tag{1}$$

Ruth (6) used an equivalent expression:

$$\frac{\Delta P}{L} = \frac{u\mu\alpha^1\rho_o}{g_c} \tag{2}$$

It is evident from Equations (1) and (2) that

$$\alpha^1\rho_o = \frac{1}{\alpha} \tag{3}$$

The Darcy equation may be adapted to centrifugal filtration as follows: For a centrifugal filter

$$\Delta P = \frac{a\rho_f H}{g_c} \tag{4}$$

where

$$a = \omega^2 r_{avg} = (2\pi N)^2 r_{avg} \tag{5}$$

and

$$H = \frac{\text{volume}}{\text{area}} = \frac{r_o^2 - r_f^2}{2r_{avg}} \tag{6}$$

Substitution of Equations (5) and (6) into Equation (4) yields

$$\Delta P = \frac{(2\pi N)^2 \rho_f (r_o^2 - r_f^2)}{2g_c} \tag{7}$$

Substituting Equation (7) into Equation (1) and noting that $u = \frac{Q_s}{A_c}$ leads to

$$\frac{Q_s}{A_c} = \frac{(2\pi N)^2 \alpha \rho_f (r_o^2 - r_f^2)}{2\mu L} \tag{8}$$

Taking A_c as

$$A_c = 2\pi r_{lm} h \tag{9}$$

and substituting in Equation (8) leads finally to the expression

$$Q_s = \frac{4\pi^3 N^2 \rho_f h \alpha (r_o^2 - r_f^2)}{\mu \ln \frac{r_o}{r_c}} \tag{10}$$

Equation (10) is the theoretical expression for the rate of flow of filtrate Q_s, of density ρ_f, and of viscosity through a bed of solids of permeability α in a centrifugal filter in which the liquid level is at r_f and the cake level is at r_c. The permeability has a constant value for an incompressible cake but changes with pressure when the cake is compressible.

The resistance of the filter medium to flow of the filtrate is not included in Equation (10). It may be taken into

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Fig. 1. Large basket.

account as

$$Q_T = \frac{4\pi^3 N^2 \rho_f h\ (r_o^2 - r_f^2)}{\mu \left[\dfrac{\ln \dfrac{r_o}{r_c}}{\alpha} + \dfrac{R_m}{r_o} \right]} \quad (11)$$

Adaptations of the Darcy equation for centrifugal filtration have been proposed by Maloney (*5*), Smith (*7*), Burak and Storrow (*1*), Inglesent and Storrow (*4*), and Grace (*2*). The Burak-Storrow and Grace equations have been supported by experimental data and are equivalent to Equations (10) and (11) respectively. From Burak and Storrow

TABLE 1. DIMENSIONS AND OPERATING RANGES OF EXPERIMENTAL CENTRIFUGAL FILTERS

	Small basket	Large basket
Diameter, D, in.	5	11
Radius to inside of cloth r_o, in.	2.303	5.187
Liquid levels r_f, in.		
Level 1	1.937	3.938
Level 2	2.047	4.469
Radius to cake surface r_c, in.	As required	As required
Depth of basket h, in.	1.125	2.125
Range of revolutions per minute	300-1,000	300-1,000
	1,200-4,000	1,200-4,000
Maximum gees	1,140	2,500
Maximum total hydraulic driving force for filtration (water) ΔP, lb.-force/sq. in.	13	93

$$Q_s = \frac{4\pi^3 N^2 K_c h (r_o^2 - r_f^2)}{\mu g \ln \dfrac{r_o}{r_c}} \quad (12)$$

According to Grace

$$Q_T = \frac{4\pi^3 N^2 \rho_f h (r_o^2 - r_f^2)}{\mu \left[\rho_s \alpha^1 (1-\epsilon) \ln \dfrac{r_o}{r_c} + \dfrac{R_m}{r_o} \right]} \quad (13)$$

Equations (10) and (12) are seen to be equivalent, as are Equations (11) and (13), from the following relationships:

$$\alpha = \frac{K_c}{\rho_f g} = \frac{1}{\rho_s \alpha^1 (1-\epsilon)} \quad (14)$$

For later use it is noted here that for additive resistances

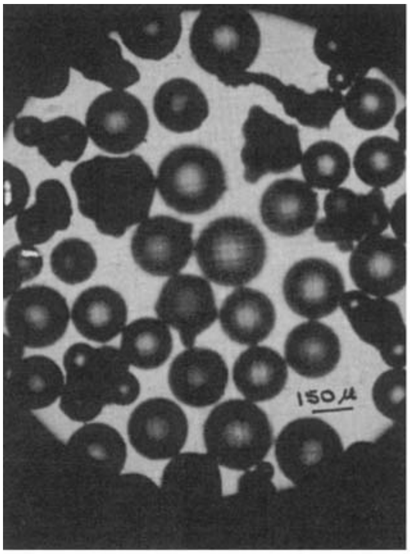
$$\frac{(\Delta P)_T}{Q_T} = \frac{(\Delta P)_m}{Q_m} + \frac{(\Delta P)_s}{Q_s} \quad (15)$$

and that

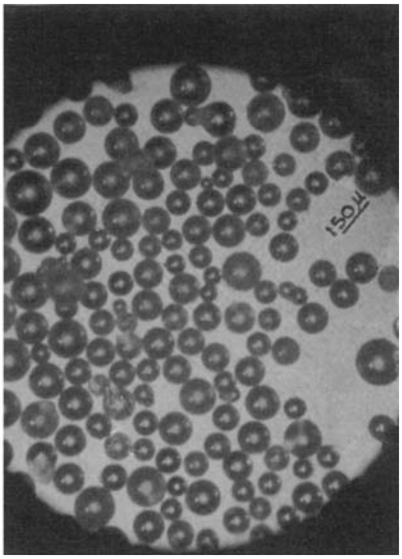
$$\frac{1}{Q_T} = \frac{1}{Q_m} + \frac{1}{Q_s} \quad (16)$$

when $(\Delta P)_T = (\Delta P)_m = (\Delta P)_s$ from separate runs.

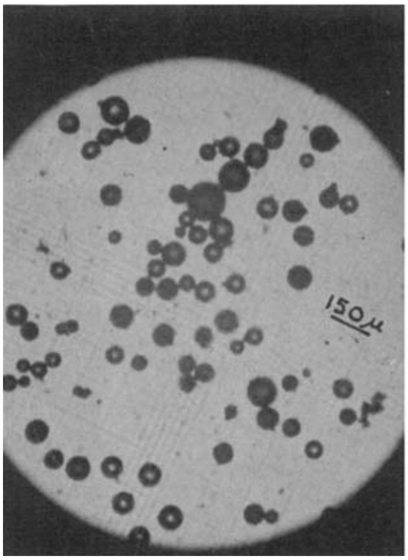
Three types of experiments are run to check the theoretical equations: (1) centrifugal drain rates where filtrate is run through a previously deposited cake, (2) centrifugal filtration rates where a slurry is fed into the machine and a constant liquid head is maintained, and (3) modified centrifugal filtration wherein cake is deposited in successive increments and drain-rate data are taken on the beds so formed. Haruni and Storrow (*3*) performed several centrifugal drain-rate runs on starch, chalk, and kieselguhr cakes and



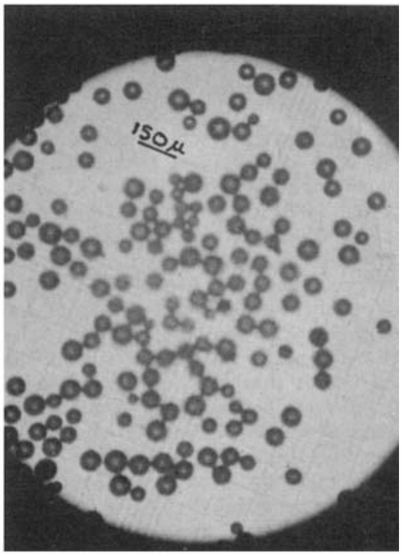
50/100 mesh



100/200 mesh



200/400 mesh



400/—mesh

Fig. 2. Photomicrographs of styrene-divinyl benzene spheres.

TABLE 2. RANGES OF VARIABLES

Revolutions per minute	Liquid depth, in.	Cake thickness, in.	Revolutions per minute squared $\times 10^{-6}$	$r_o^2 - r_f^2$, sq. in.	$(\Delta P)_T$, lb./sq. in.	Re	Particle size, mesh	Viscosity, cp.
5-in.-diameter Basket								
1,330-3,810	0.256, 0.366	0.069-0.233	1.8-14.5	1.11, 1.55	1.0-11.5	0.01-1.7	50/100, 100/200 200/400, 400/–	0.81-0.91
11-in.-diameter Basket								
1,310-2,550	0.718, 1.249	0.380-0.633	1.7-6.5	6.93, 11.4	6.1-38	0.02-0.7	50/100 200/400, 400/–	0.85, 2.2

found agreement with the revolutions per minute, liquid depth ($r_o^2 - r_f^2$), and viscosity factors of Equation (12). Earlier attempts by Burak and Storrow (*1*) and Inglesent and Storrow (*4*) had been unsuccessful. General agreement was obtained by Grace (*2*) with the revolutions per minute and cake depth [$\ln(r_o/r_c)$] factors in drain rate and modified filtration runs on cakes of titanium dioxide, carbon, cellulose fibers, or diatomaceous earth. However values of 0.5 to 1.7 were found for the exponent of N, some of the cakes apparently being compressible. The expected value of 2 was not found, however, even with incompressible cakes. Concurrently with the work undertaken at Louisiana State University and reported in this paper, Valleroy (*8*) at the University of Kansas obtained additional support for the revolutions per minute and liquid-depth factors with drain-rate data taken on Lucite spheres.

This paper reports the data taken in twenty-five centrifugal drain-rate runs on four sizes of styrene-divinylbenzene spheres. Equation (10) is applied to the data by statistical analysis.

EQUIPMENT, METHOD OF OPERATION, MATERIALS

Equipment

Two perforated baskets for centrifugal filtration were mounted horizontally on individual drive shafts. Figure 1 shows the larger basket. By means of a variable-speed drive and two sets of pulleys, it was possible to drive either basket over the approximate ranges from 300 to 1,000 and 1,200 to 4,000 rev./min. Thus in the 5-in. –and 11-in.–diameter baskets it was possible to develop maximum centrifugal forces of approximately 1,140 and 2,500 Gees, respectively. Revolutions per minute were measured with a stroboscope.

Water or other filtrate was run into the small basket through a spray nozzle and into the large basket through a wing tip. A glycerol-water solution was supplied by means of a centrifugal pump and storage tank. Flow rates were regulated by a hand-operated needle valve.

Dimensions and operating ranges are given in Table 1 for both baskets.

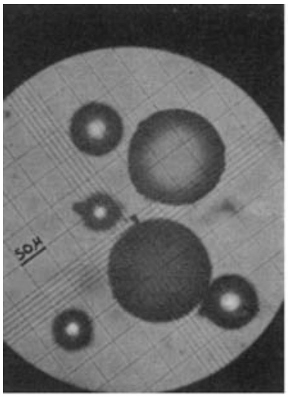
From the dimensions given in Table 1 and Equation (7) the maximum driving force for the filtration of water through porous cakes is calculated to be approximately 13 lb./sq.in for the small bowl and 93 lb./sq.in. for the large one. Approximately 1,140 and 2,500 Gees, respectively, can be developed. Thus, on this laboratory installation it is possible to cover the range of Gees and pressure drop existing in normal commercial operation. For example, a 40-in.–diameter commercial machine operating at 800 rev./min. and a liquid depth of 6 in. would develop 364 Gees at the basket wall and a total hydraulic driving force of 67 lb./sq.in. when the filtrate has the density of water.

Cloth, supported by coarse wire mesh, served as the septum. Putty and caulking were forced into seals at both the back and the front of the basket to prevent leaks of filtrate around the ends of the cake. It was possible to make several runs with one cloth.

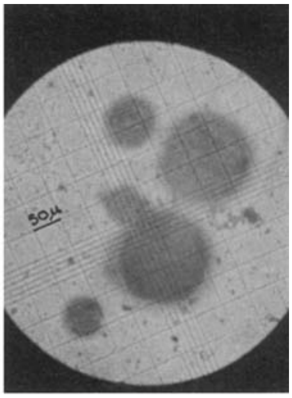
Method of Operation

Cake was deposited by spooning dry solids into a pool of liquid in the rotating basket. The mass of cake used for the small bowl was 10, 20, or 25 g., whereas 300 to 450 g. were used in the large machine. In the case of the small bowl the cake was deposited in approximately 1-g-increments, whereas the increments were 5 to 10 g. for the large bowl. This procedure forms a smooth and uniform cake and simulates the actual formation of a cake when slurry is fed and there is a liquid level above the cake. Any effect of classification is minimized by addition of the solids in many increments. A liquid level was maintained above the cake at all times during a run.

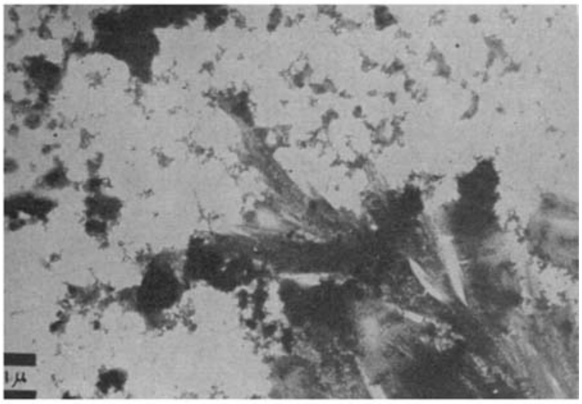
The flow rate of filtrate was measured over a range of revolutions per minute and at each of two liquid levels for each cake deposited as above. Immediately before each cake run the flow rate of filtrate through the septum only was measured over the same range of revolutions per



200/400 mesh



fines from 200/400 mesh



electronmicrograph, fines from 200/400 mesh

Fig. 3. Photomicrographs of styrene-divinyl benzene spheres.

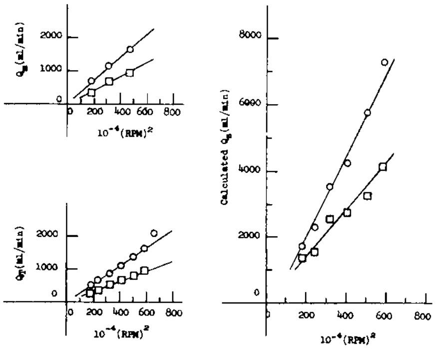


Fig. 4. Typical data (run 22), large basket, water. 400/—mesh styrene-divinyl benzene spheres.

r_f in.	Symbol
3.938	○
4.469	□

minute and at the same two liquid levels.

Materials

Styrene-divinylbenzene spheres comprised the cakes. Four mesh ranges were used as received from the manufacturer: 50/100, 100/200, 200/400, and 400/— mesh. Photomicrographs of these materials and the fines contained in them are shown in Figures 2 and 3.

DATA

The ranges of variables over which the data* were taken on the styrene-divinylbenzene spheres are shown in Table 2.

Data for a typical run (run 22) are plotted in Figure 4. As predicted by Equations (10) through (13), for an incompressible bed (constant permeability) the flow rate is proportional to revolutions per minute squared. By Equation (16) the flow rate which would exist through the solids if there were no resistance in the septum may be calculated. Plots for Q_s so calculated for run 22 are also given in Figure 4.

Q_s was calculated for individual data points for all runs. These calculated values of Q_s as well as the primary data Q_m and Q_T were used to check the theoretical correlation. To check N and the liquid-depth factor $(r_o^2 - r_f^2)$, the data for each cake were fitted to the model

$$Q = A\,N^b (r_o^2 - r_f^2)^c \qquad (17)$$

by the method of least squares. Table 3 summarizes the results.

Runs 11, 12, 23, and 24 were not applicable to testing Equation (17) because they contained only two data points, whereas a three-constant equation was being evaluated. The exponents calculated from Q_T and Q_s for runs 8, 13, and 16 lay outside the 1% confidence limits and therefore were excluded from the mean values. As might be expected, the precision in the values of the exponents is best where the only resistance to flow is made up by the filter medium. Greater variability of data is seen when filter medium and cake comprise the resistance to flow, and as a result of being calculated from Q_m and Q_T the values of the exponents predicted from Q_s show the greatest variability of all.

Table 3. Exponents for N and $(r_o^2 - r_f^2)$

Calculated from	Number of runs	Exponent	Mean	Standard deviation Single value	Standard deviation Mean
Q_m	21	b	2.04	0.15	0.03
		c	1.13	0.31	0.07
Q_T	18	b	2.03	0.33	0.08
		c	1.22	0.35	0.08
Q_s	18	b	1.98	0.64	0.15
		c	1.23	0.60	0.14

From the mean values of the exponents listed in Table 3 and their standard deviations it is concluded that the data for any particular cake may be correlated satisfactorily by the expression

$$Q_s = AN^2(r_o^2 - r_f^2) \qquad (18)$$

as predicted by the theoretical Equation (10).

To check the cake-thickness factor [ln (r_o/r_c)] from run to run, it is necessary to have cakes of constant characteristics, that is the same permeability. Consequently the permeability was calculated by Equation (10) for all readings on all cakes. The averages for each cake run are listed in Table 4 except for runs 8, 13, and 16, which were eliminated above from further consideration.

If the cakes were formed reproducibly and if Equation (10) describes the flow through them, then the same permeability should be calculated for all runs in a given mesh size. However wide variation is observed in the calculated permeabilities, with the possible exception of those for 100/200 mesh particles. This general lack of reproducibility may have been caused by differing structure in the cakes (perhaps owing to the fines), by experimental difficulties such as in measuring r_c sufficiently accurately, or possibly by the inadequacy of Equation (10).

The runs on 100/200 mesh particles show the smallest variation in the calculated permeabilities, and an attempt was made to check the effect of the cake-thickness factor here. However the correlation coefficient of A [Equation (17)] plotted vs. the reciprocal of ln (r_o/r_c) is only 0.475, which constitutes relatively weak statistical support for the theoretical equation. Therefore it is concluded that other more suitable data must be taken to confirm the cake-thickness factor.

The viscosity was varied significantly for run 25, in which a glycerol-water solution served as the filtrate. But the great variation in the apparent permeabilities makes it impossible to compare run 25 with the other runs on 400/— mesh material and therefore impossible to check the effect of viscosity.

Table 4. Permeabilities of Cakes of Styrene-DVB Spheres

Run	Radius to cake, in. surface	Viscosity, cp	Permeability, $\alpha \times 10^{18}$, ft.^{2} Mean	s of Mean
50/100 Mesh				
1	2.191	0.85	9.0	0.7
2	2.184	0.85	6.8	0.6
3	2.070	0.85	37.2	2.3
4	2.156	0.828	11.5	1.1
5	2.223	0.819	9.6	1.2
6*	4.554	0.85	20.3	0.9
100/200 Mesh				
7	2.188	0.85	5.5	0.7
8	2.188	0.838	eliminated	
9	2.164	0.819	8.9	1.0
10	2.164	0.838	6.1	0.4
11	2.196	0.814	5.8	1.2
12	2.185	0.814	6.4	1.0
23	2.195	0.907	5.1	0.5
24	2.196	0.907	3.8	0.3
200/400 Mesh				
13	2.195	0.85	eliminated	
14	2.188	0.85	12.2	1.5
15	2.230	0.85	6.0	0.6
16*	4.651	0.85	eliminated	
400/— Mesh				
17	2.195	0.828	4.6	0.4
18	2.188	0.838	11.0	0.8
19	2.230	0.866	6.4	0.4
20	2.183	0.828	13.0	1.0
21	2.234	0.828	4.5	0.3
22*	4.807	0.866	10.1	0.3
25†	2.177	2.19	27.4	2.2

* Large basket used for these runs, small basket for all others.
† Glycerol-water solution used for filtrate, all others water.

* Data and calculated results are on file at University Microfilms, 313 North First Street, Ann Arbor, Michigan, as Publication No. 21,982, Mic. 57-3157.

APPENDIX B

LEAST SQUARES FIT OF DATA TO MODEL $Q = A\,N^b(r_o^2 - r_f^2)^c$

Run	Q_m −log A	b	c	$\left[\begin{smallmatrix}\text{Correlation}\\ \text{coefficient}\end{smallmatrix}\right]^2$	Q_T −log A	b	c	$\left[\begin{smallmatrix}\text{Correlation}\\ \text{coefficient}\end{smallmatrix}\right]^2$	Q_s −log A	b	c	$\left[\begin{smallmatrix}\text{Correlation}\\ \text{coefficient}\end{smallmatrix}\right]^2$
50/100 Mesh												
1	4.419	2.008	1.352	0.99	4.710	2.046	1.028	0.98	4.460	2.120	0.463	0.86
2	5.416	2.310	1.227	0.99	4.889	2.063	1.606	0.99	3.519	1.743	2.090	0.98
3	3.196	1.912	0.577	0.99	5.854	2.580	0.830	0.98	6.692	2.891	0.958	0.97
4	3.468	1.765	1.294	0.99	2.873	1.504	1.498	0.96	1.593	1.222	1.716	0.86
5	4.464	1.937	1.611	0.99	5.045	2.085	1.454	0.99	6.248	2.679	0.696	0.86
6	3.630	1.894	0.850	0.99	3.415	1.756	0.876	0.99	2.327	1.538	0.912	0.99
100/200 Mesh												
7	5.265	2.250	1.258	0.99	4.656	1.985	1.885	0.86	3.568	1.710	2.425	0.66
8	4.435	2.029	1.126	0.96	4.983	2.045	2.147	0.97	4.780	2.031	2.941	0.94
9	4.218	1.868	1.587	0.99	3.279	1.551	1.370	0.99	0.235	0.826	0.859	0.82
10	4.609	2.112	1.640	0.99	5.603	2.276	1.136	0.99	5.763	2.381	1.790	0.96
11	Not enough data				Not enough data				Not enough data			
12	Not enough data				Not enough data				Not enough data			
23	Not enough data				Not enough data				Not enough data			
24	Not enough data				Not enough data				Not enough data			
200/400 Mesh												
13	4.965	2.141	1.296	0.98	9.907	3.349	2.307	0.99	10.909	3.662	2.570	0.99
14	4.033	2.032	0.728	0.99	6.652	2.703	0.888	0.99	8.371	3.290	1.004	0.99
15	4.564	2.097	0.870	0.99	5.364	2.256	0.992	0.97	5.607	2.431	1.136	0.87
16	4.251	1.985	0.842	0.99	9.751	3.358	1.553	0.99	14.455	4.683	2.249	0.99
400/– Mesh												
17	4.246	1.918	1.030	0.98	4.292	1.845	1.413	0.95	3.666	1.751	1.904	0.88
18	4.528	2.038	1.256	0.99	5.026	2.140	1.139	0.98	5.148	2.333	0.833	0.85
19	5.587	2.298	1.283	0.99	5.274	2.165	1.330	0.99	3.284	1.742	1.497	0.95
20	4.990	2.192	1.180	0.98	4.574	2.024	1.018	0.99	3.117	1.754	0.642	0.88
21	5.103	2.177	0.926	0.99	5.128	2.124	1.232	0.99	4.435	2.046	1.871	0.87
22	4.584	1.996	1.096	0.99	4.773	2.033	1.044	0.99	4.303	2.128	0.884	0.95
25	4.001	1.955	0.635	0.99	2.815	1.511	0.559	0.98	0.686	0.966	0.450	0.81

CONCLUSIONS

Centrifugal filtration data taken on four mesh sizes of styrene-divinylbenzene spheres support the theoretical form of the revolutions per minute and liquid-depth factors in the adapted Darcy equation. Indices of 2 and 1, respectively, are indicated. The data do not allow an adequate check of the cake-thickness and viscosity factors because of a general irreproducibility of cakes.

ACKNOWLEDGMENT

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NOTATION

A = constant for incompressible cake
A_c = area of cake perpendicular to flow
a = acceleration in centrifugal field
b = constant
c = constant
g_c = conversion factor (32.2 lb.-mass/lb.-force-ft./sec.^{2})
h = depth of centrifugal basket parallel to the axis
H = depth of liquid in centrifugal filter
K_c = cake permeability (lb.-mass/sec.^{2})
L = depth of bed in direction of flow
L_m = thickness of filter medium
N = rate of rotation of centrifugal basket
P = pressure
$(\Delta P)_T$ = total pressure drop across cake and medium
$(\Delta P)_c$ = pressure drop across cake
$(\Delta P)_m$ = pressure drop across medium
Q_m = filtrate flow rate through medium only
Q_s = filtrate flow rate through solids only
Q_T = filtrate flow rate through medium and cake
Re = Reynolds number
R_m = filter medium resistance (ft.$^{-1}$ = L_m/α_m)
r = radius
r_{avg} = radius to average depth of liquid
r_c = radius to surface of cake
r_f = radius to surface of fluid
r_{lm} = log-mean radius
r_o = radius to inside of filter medium
u = superficial velocity of filtrate

Greek Letters

α = cake permeability, (ft.^{2}) property of the cake
α_m = permeability of filter medium
α^1 = specific resistance of cake (ft./lb.-mass)
Δ = finite change of variable
ϵ = cake porosity
μ = absolute viscosity
ρ_c = bulk density of cake
ρ_f = density of fluid
ρ_s = true density of solids comprising cake
ω = angular acceleration

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